

General Physics I with Calculus.

↓
Broad

↓
Derivative & Integration

e.g. constant $a \rightarrow \vec{a}(t)$

$$v = u + at \rightarrow \vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}(t') dt'$$

Study of natural phenomena in terms of matters and their interactions.

e.g. gravity:
↓
projectile motion
orbital motion

↓
mass

↓
gravitational
attraction

Real world is complicated $\rightarrow$ model is needed.

Reality $\rightarrow$ model $\rightarrow$ theory $\rightarrow$ prediction.

↓
Keep important
features/factors
only.

SI unit

Fundamental Quantities (Dimension)

Time (T)

Length (L)

Mass (M)

Temperature

Charge ...

units:

day, month...

foot, yard...

oz, pound...

'C', 'F

...

SI unit:

second (s)

meter (m)

kilogram.

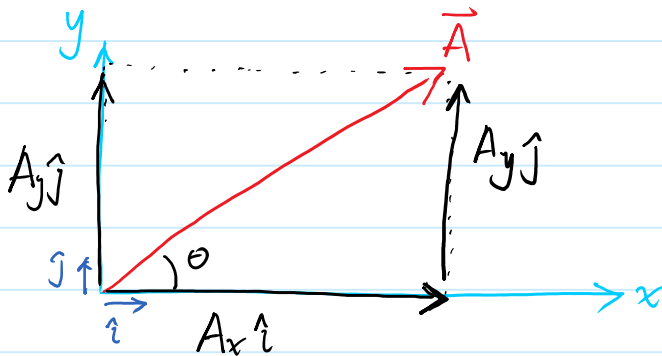
K.

...

Vector

"arrow" $\begin{cases} \text{length / magnitude} \\ \text{direction} \end{cases}$

2D Cartesian Coordinate



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

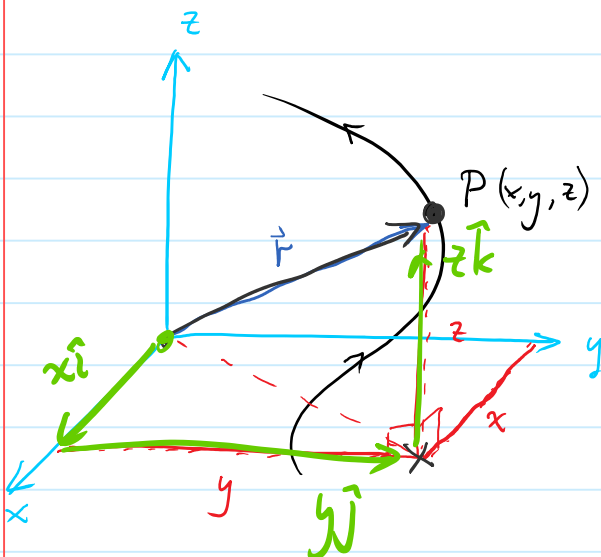
$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\Rightarrow \tan \theta = \frac{A_y}{A_x}$$

$$\begin{cases} A = \sqrt{A_x^2 + A_y^2} \\ \tan \theta = A_y / A_x \end{cases} \Leftrightarrow \begin{cases} A_x = A \cos \theta \\ A_y = A \sin \theta \end{cases}$$

$$\vec{A} = (A_x, A_y)$$

Kinematics in 3D - trajectory of a particle in vector notation



position vector : $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

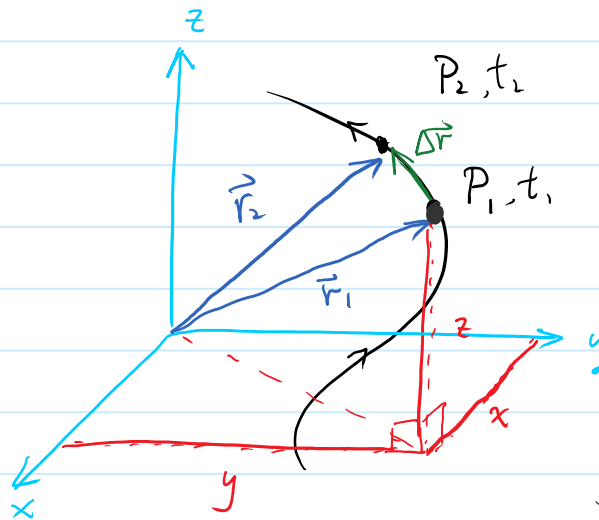
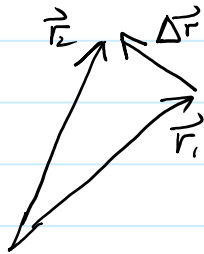
x, y, z are the coordinates of the particle

They can be a function of time :

$$\vec{r}(t) = \underline{x(t)}\hat{i} + \underline{y(t)}\hat{j} + \underline{z(t)}\hat{k}$$

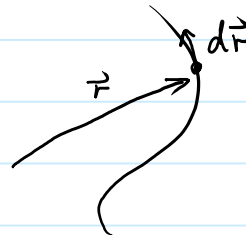
Displacement:

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$



Infinitesimal displacement:

$d\vec{r}$ is always tangential to the trajectory.

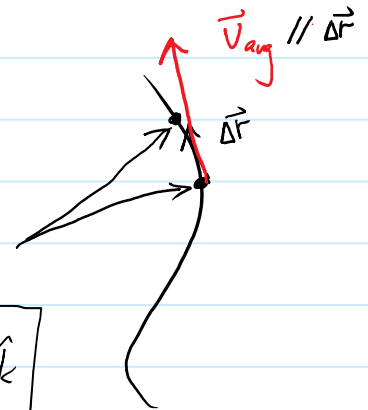


Velocity

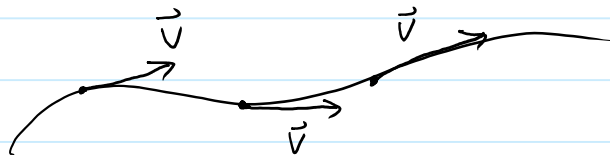
$$\text{average velocity} = \vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\text{instantaneous velocity} = \vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$$

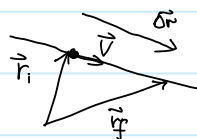
$$\text{or } \boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}}$$



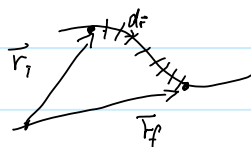
$\vec{v}$ is tangential to trajectory at any point.



To find $\vec{r}$ from $\vec{v}$,



$$\Delta \vec{r} = \vec{v} \Delta t = \vec{v}(t_f - t_i)$$



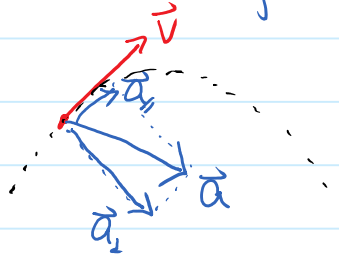
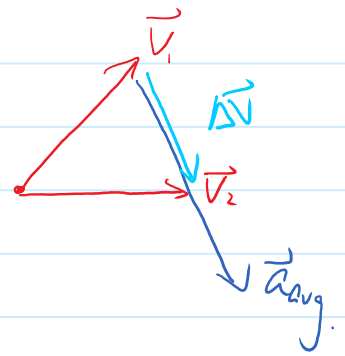
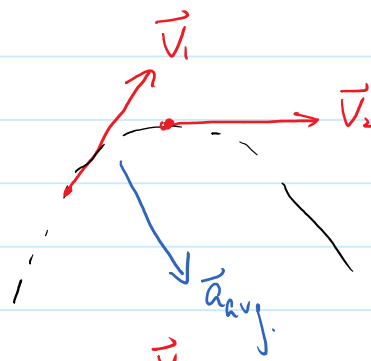
$$d\vec{r} = \vec{v} dt$$

$$\vec{r}_f - \vec{r}_i \equiv \Delta \vec{r} = \int d\vec{r} = \int_{t_i}^{t_f} \vec{v} dt$$

Acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\rightarrow \vec{a} = \frac{d\vec{v}}{dt}$$

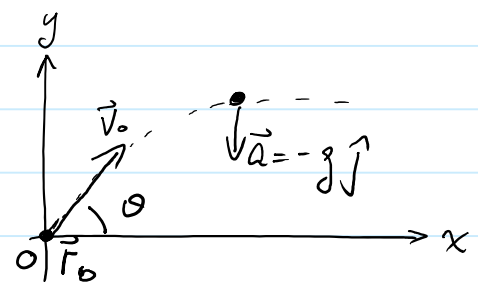


For every $\vec{a}$, we decompose it into 2 vectors

- ① $\vec{a}_t \parallel \vec{v}$, $\vec{a}_t = a_t \hat{v}$, $a_t = \frac{d|\vec{v}|}{dt}$ rate of change of speed.
- ② $\vec{a}_c \perp \vec{v}$, $|\vec{a}_c| = \frac{v^2}{R}$ centripetal acc.
change direction only

Projectile motion

Given initial position $\vec{r}_0$ and initial velocity $\vec{v}_0$



$$\vec{a} = -g\hat{j}$$

find $\vec{v}(t)$ and $\vec{r}(t)$.

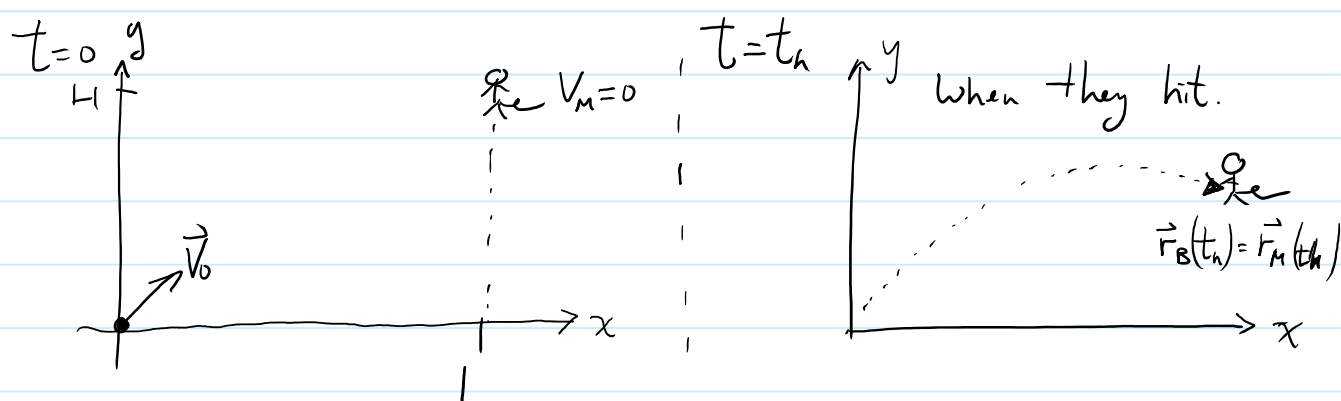
$$\vec{v}_0 = v_0 \cos \theta \hat{i} + v_0 \sin \theta \hat{j}$$

$$\vec{r}_0 = \vec{0}$$

$$\begin{aligned}\vec{V}(t) &= \vec{V}(0) + \int_0^t \vec{a}(t') dt' \\ &= V_0 \cos \theta \hat{i} + V_0 \sin \theta \hat{j} + \int_0^t -g dt' \hat{j} \\ \boxed{\vec{V}(t) &= V_0 \cos \theta \hat{i} + (V_0 \sin \theta - gt) \hat{j}}\end{aligned}$$

$$\begin{aligned}\vec{r}(t) &= \vec{r}(0) + \int_0^t \vec{V}(t') dt' \\ &= \vec{0} + \int_0^t V_0 \cos \theta dt' \hat{i} + \int_0^t (V_0 \sin \theta - gt') dt' \hat{j} \\ \boxed{\vec{r}(t) &= V_0 \cos \theta t \hat{i} + (V_0 \sin \theta t - \frac{1}{2}gt^2) \hat{j}}\end{aligned}$$

Example Monkey and Hunter



Position vector of bullet's trajectory.

$$\vec{r}_B(t) = V_0 \cos \theta t \hat{i} + (V_0 \sin \theta t - \frac{1}{2}gt^2) \hat{j}$$

Position vector of monkey's trajectory.

$$\vec{r}_M(t) = L \hat{i} + (H - \frac{1}{2}gt^2) \hat{j}$$

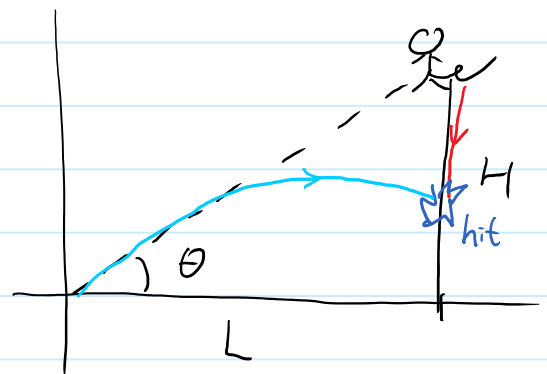
To hit, $\exists t_h$ s.t. $\vec{r}_M(t_h) = \vec{r}_B(t_h)$

$$\Rightarrow L \hat{i} + \left(H - \frac{1}{2} g t_h^2 \right) \hat{j} = V_0 \cos \theta t_h \hat{i} + \left(V_0 \sin \theta t_h - \frac{1}{2} g t_h^2 \right) \hat{j}$$

$$\Rightarrow \begin{cases} L = V_0 \cos \theta t_h \\ H - \cancel{\frac{1}{2} g t_h^2} = V_0 \sin \theta t_h - \cancel{\frac{1}{2} g t_h^2} \end{cases}$$

$$\Rightarrow \begin{cases} L = V_0 \cos \theta t_h \\ H = V_0 \sin \theta t_h \end{cases}$$

$$\Rightarrow \tan \theta = \frac{H}{L}$$



Dimension

in equation

Dimension on both sides of a equality must be the same.

e.g. $A = \pi r^2$

$L^2 = 1 \times L^2$

Dimensionless

$$A = \pi^2 r$$

$L^2 \neq L$

$$[A] = [\pi] [r^2]$$

$$\Rightarrow L^2 = 1 \times L^2$$

in elementary function

$$[\sin x] = 1 \quad \text{dimensionless}$$

so as the argument : $[x] = 1$

e.g. Suppose in oscillating motion

$$x(t) = A \sin(\omega t) \Rightarrow [\omega t] = 1$$

$$[\omega] = \frac{1}{[t]} = \frac{1}{T} = \text{frequency.}$$

Uncertainty and Significant Figure

Every value obtained from measurement has error/uncertainty.

Precision of the value is represented by its significant figure.

e.g. $T_V = 3.14 \rightarrow \begin{array}{c} \text{range} \\ 3.135 \text{ to } 3.145 \\ \text{or } 3.135 \leq x < 3.145 \end{array}$

$$T_V = 3.1416 \rightarrow 3.14155 \leq x < 3.14165$$

Experiment

$$\begin{array}{c} 2.017676 \pm 0.0132 \\ \swarrow \\ 2.017676 \pm 0.013 \\ \text{not important.} \quad \text{3 d.p.} \end{array}$$

keep 1 or 2 sig. fig.

$$\Rightarrow \boxed{2.018 \pm 0.013}$$

Multiplication: $\frac{0.745 \times \underline{2.2}}{3.885} = 0.42$ to smallest sig. fig.

Addition / : $27.153 + 138.\underline{2} - 11.74$

Subtraction
 $= 153.6$ to the least d.p.
(the greatest place value.)